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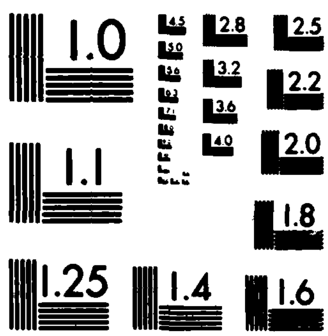

MICROCOPY RESOLUTION TEST CHART

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A Lower Confidence Bound
on the Probability of a Correct Selection*

by

Woo-Chul Kim
Seoul National University and Purdue University

Technical Report #85-18

PURDUE UNIVERSITY

DEPARTMENT OF STATISTICS

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ABSTRACT

In the problem of selecting the best of k populations, a natural rule is to select the population corresponding to the largest sample value of an appropriate statistic. As a retrospective analysis, a lower confidence bound on the probability of a correct selection is derived when the probability density function has the monotone likelihood ratio property under the location parameter setting. The result is applied to the normal populations with both known and unknown common variance. Tables to implement the confidence bound are provided. Additional keywords: Charts; tables (data).

KEY WORDS: Selection problem; A retrospective analysis; Probability of a correct selection; Lower confidence bound; Monotone likelihood ratio.

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1. INTRODUCTION

Consider independent observations X_{ij} from each of k populations with cdf's $G(x-\theta_i)$, $i=1,2,\ldots,k$, $j=1,2,\ldots,n$. The experimenter wishes to select the "best" population associated with the largest parameter θ_i. For this purpose, we choose an appropriate statistic $Y_i=Y(X_{i1},\ldots,X_{in})$ with cdf $F_n(y-\theta_i)$ and use the natural selection rule which selects the population corresponding to the largest Y_i as the best.

For this selection problem, Bechhofer (1954) introduced the indifference zone approach in which we determine the sample size n, prior to the experiment, to control the probability of a correct selection (PCS)

$$PCS = \int_{-\infty}^{\infty} \prod_{i=1}^{k-1} F_n(Y+\theta_{[k]}-\theta_{[i]})\, d\, F_n(y) \tag{1.1}$$

where $\theta_{[1]} \leq \theta_{[2]} \leq \cdots \leq \theta_{[k]}$ are the ordered parameters. In controlling the PCS, we need to specify a preference zone where the largest two parameters $\theta_{[k]}$ and $\theta_{[k-1]}$ are far apart. This indifference zone approach is clearly formulated from the point of view of designing experiment.

Recently retrospective analyses regarding the PCS have been studied by Gibbons, Olkin, and Sobel (1977), Anderson, Bishop, and Dudewicz (1977),

Olkin, Sobel, and Tong (1982), and Faltin and McCulloch (1983) among others. Most of these studies have dealt with the point estimation of the PCS.

Gibbons, Olkin, and Sobel (1977) and Olkin, Sobel, and Tong (1982) have presented interval estimates of PCS. However the coverage probabilities of such interval estimates have not be discussed. Thus they can not be interpreted as confidence interval estimates (see Bechhofer 1980, p. 753). In the case of normal populations, Anderson, Bishop, and Dudewicz (1977) have given a lower confidence bound on PCS. The quantile unbiased estimator in Faltin (1980) can also be regarded as a lower confidence bound on PCS. However, it is restricted to the special case of k=2 populations.

This article presents a lower confidence bound on PCS when the pdf $f_n(y-\theta)$ of $F_n(y-\theta)$ has the monotone likelihood ratio (MLR) in y and θ. From this result, we obtain a lower confidence bound on PCS in the case of normal populations with both known and unknown common variance. The obtained lower confidence bound is sharper than that of Anderson, Bishop, and Dudewicz (1977), and reduces to that of Faltin (1980) in the special case of k=2 populations. Tables to implement the lower confidence bound as well as an illustrative example are given.

2. A LOWER CONFIDENCE BOUND ON PCS

It can be easily seen from the inquality

$$PCS \geq \int_{-\infty}^{\infty} F^{k-1}(y + \theta_{[k]} - \theta_{[k-1]})\, d\, F(y) \qquad (2.1)$$

that a (conservative) lower confidence bound on PCS can be obtained from a lower confidence bound on $\theta_{[k]} - \theta_{[k-1]}$. Thus we begin with constructing a

lower confidence bound on $\theta_{[k]}-\theta_{[k-1]}$. To do this, let $Y_{(1)}\leq Y_{(2)}\leq\cdots\leq Y_{(k)}$ denote the ordered statistics of $Y_1,Y_2,\ldots,Y_k$, and let $f(y)$ denote the pdf of F. Note that the dependence of F and f on n is suppressed notationally.

We first state a lemma which is a generalization of a result in Anderson, Bishop, and Dudewicz (1977).

Lemma 1. Assume that $\log f(y)$ is concave. Then for any fixed $c>0$, $P_{\underset{\sim}{\theta}}[Y_{(k)}-Y_{(k-1)}>c]$ is non-increasing in $\theta_{[1]}$.

Proof. By semmetry we may assume $\theta_1 \leq\ldots\leq \theta_k$. Then, for any $c>0$, we have

$$P_{\underset{\sim}{\theta}}[Y_{(k)}-Y_{(k-1)}>c] = \sum_{j=1}^{k} \int_{-\infty}^{\infty} \prod_{\substack{i=1\\ i\neq j}}^{k} F(y+\theta_j-\theta_i-c)\ f(y)\ dy.$$

Therefore,

$$\frac{\partial}{\partial\theta_1} P_{\underset{\sim}{\theta}}[Y_{(k)}-Y_{(k-1)}>c]$$

$$= \sum_{j=2}^{k} \int_{-\infty}^{\infty} \prod_{\substack{i=2\\ i\neq j}}^{k} F(y+\theta_1-\theta_i-c)\ f(y+\theta_1-\theta_j-c)\ f(y)\ dy\ -$$

$$\sum_{j=2}^{k} \int_{-\infty}^{\infty} \prod_{\substack{i=2\\ i\neq j}}^{k} F(y+\theta_j-\theta_i-c)\ f(y+\theta_j-\theta_1-c)\ f(y)\ dy$$

$$= \sum_{j=2}^{k} \int_{-\infty}^{\infty} \prod_{\substack{i=2\\ i\neq j}}^{k} F(y-\theta_i-c)\ [f(y-\theta_j-c)\ f(y-\theta_1) - f(y-\theta_1-c)\ f(y-\theta_j)]\ dy.$$

By the equivalence between the assumption and the MLR of $f(y-\theta)$ in y and θ, the expression in the brackets is non-positive. Hence the result follows.

To define a lower confidence bound, let

$$H(x) = \int_{-\infty}^{\infty} F(x+y)\, f(y)\, dy$$

denote the cdf of $(Y_1-\theta_1) - (Y_2-\theta_2)$ and let $x_{\alpha/2}$ denote the upper $\alpha/2$ quantile of $H(x)$ for $0<\alpha<1$. Note that $H(x)$ is symmetric and $x_{\alpha/2}>0$. For a given $0<\alpha<1$ and for $t \geq x_{\alpha/2}$, we define a non-negative function $L_\alpha(t) = L(t)$ by

$$H(L(t)-t) + H(-L(t)-t) = \alpha. \tag{2.2}$$

The existence of such a function $L(t)$ for $t \geq x_{\alpha/2}$ is proved in the Appendix under the assumption in Lemma 1. Also it can be easily observed that the function $L(t)$ is strictly increasing for $t \geq x_{\alpha/2}$.

We present an exact $100(1-\alpha)\%$ lower confidence bound on $\theta_{[k]}-\theta_{[k-1]}$ in the following theorem.

<u>Theorem 1</u>. Assume that $\log f(y)$ is concave. Then

$$\inf_{\underset{\sim}{\theta}} P_{\underset{\sim}{\theta}}[\theta_{[k]}-\theta_{[k-1]} \geq L(Y_{(k)}-Y_{(k-1)})] = 1-\alpha \tag{2.3}$$

where $L(t)$ is defined by (2.2) for $t \geq x_{\alpha/2}$ and 0 for $0 \leq t \leq x_{\alpha/2}$.

<u>Proof</u>. For any fixed $\theta_{[k]}$ and $\theta_{[k-1]}$, let $\Delta = \theta_{[k]}-\theta_{[k-1]}$. Then it follows from Lemma 1 that for all $\underset{\sim}{\theta}$

$$P_{\theta}[\Delta \geq L(Y_{(k)}-Y_{(k-1)})]$$

$$= P_{\theta}[L^{-1}(\Delta) \geq Y_{(k)}-Y_{(k-1)}]$$

$$\geq P_{\theta}[L^{-1}(\Delta) \geq |Y_{[k]}-Y_{[k-1]}|]$$

where $L^{-1}(0)$ is taken as $x_{\alpha/2}$. Note that the equality can be attained when $\theta_{[1]} = \theta_{[2]} = \ldots = \theta_{[k-2]} = -\infty$. Furthermore, for any value of Δ, we have

$$P_{\theta}[L^{-1}(\Delta) \geq |Y_{[k]}-Y_{[k-1]}|]$$

$$= 1 - \{H(\Delta-L^{-1}(\Delta)) + H(-\Delta-L^{-1}(\Delta))\}$$

$$= 1 - \alpha$$

which completes the proof.

A simple but useful corollary to Theorem 1 is the following.

Corollary 1. Under the assumption of Theorem 1, we have

$$P_{\theta}[PCS \geq \hat{P}_L] \geq 1 - \alpha \qquad \text{for all } \theta$$

where

$$\hat{P}_L = \int_{-\infty}^{\infty} F^{k-1}(y+L(Y_{(k)}-Y_{(k-1)}))\, d\, F(y). \tag{2.4}$$

The lower confidence bounds in (2.3) and (2.4) become trivial when $Y_{(k)}-Y_{(k-1)} \leq x_{\alpha/2}$, that is, when the data do not show significant evidence for $\theta_{[k]} > \theta_{[k-1]}$. In fact, it can be easily shown that any lower confidence bound for $\theta_{[k]}-\theta_{[k-1]}$, which is a non-negative and non-decreasing function of $Y_{(k)}-Y_{(k-1)}$, becomes trivial in such a case.

3. NORMAL POPULATIONS WITH A COMMON VARIANCE

Let X_{ij} be independent observations from $N(\mu_i,\sigma^2)$, $i=1,\ldots,k$, $j=1,\ldots,n$, where the common variance $\sigma^2>0$ may be either known or unknown. The best population is the one associated with $\mu_{[k]} = \max_{1\leq i\leq k} \mu_i$, and we select the population corresponding to the largest sample mean $\bar{X}_i$ as the best. The probability of a correct selection is

$$PCS = \int_{-\infty}^{\infty} \prod_{i=1}^{k-1} \Phi(x+\sqrt{n}(\mu_{[k]}-\mu_{[i]})/\sigma)d\Phi(x) \tag{3.1}$$

where $\mu_{[1]} \leq\cdots\leq \mu_{[k]}$ are the ordered μ_i's and Φ is the standard normal cdf.

In the case of known variance, by taking $Y_i = \sqrt{n}\,\bar{X}_i/\sigma$ and $\theta_i = \sqrt{n}\,\mu_i/\sigma$ in Theorem 1 and Corollary 1, we can make the following statement with 100 $(1-\alpha)$% confidence;

$$(\mu_{[k]}-\mu_{[k-1]})/\sigma \geq \sqrt{2/n}\, h(\sqrt{n}\,(\bar{X}_{(k)}-\bar{X}_{(k-1)})/\sqrt{2}\,\sigma)$$

$$PCS \geq \int_{-\infty}^{\infty} \Phi^{k-1}\,[x+\sqrt{2}h(\sqrt{n}(\bar{X}_{(k)}-\bar{X}_{(k-1)})/\sqrt{2}\,\sigma)]\,d\,\Phi(x) \tag{3.2}$$

where $\bar{X}_{(1)} \leq\cdots\leq \bar{X}_{(k)}$ are the ordered sample means and the non-negative function $h(t)$ is defined by

$$\Phi(h(t)-t) + \Phi(-h(t)-t) = \alpha \tag{3.3}$$

for $t \geq z_{\alpha/2}$ and $h(t) = 0$ for $0 \leq t \leq z_{\alpha/2}$. Here, $z_{\alpha/2}$ is the upper $\alpha/2$ quantile of the standard normal distribution.

When the common variance σ^2 is unknown, let S^2 denote the pooled sample variance. Note that $\nu S^2/\sigma^2$ has a χ^2 distribution with $\nu = k(n-1)$ degrees of freedom. Since S^2 is independent of $\overline{X}_1, \ldots, \overline{X}_k$, this case can be treated similarly by considering the conditional coverage probability given $S^2 = s^2$ and by taking $Y_i = \sqrt{n}\,\overline{X}_i/\sigma$, $\theta_i = \sqrt{n}\,\mu_i/\sigma$. Therefore we omit the derivation for the following confidence statement; With 100 $(1-\alpha)$% confidence, we have

$$(\mu_{[k]}-\mu_{[k-1]})/\sigma \;\geq\; \sqrt{2/n}\; h_\nu\; (\sqrt{n}\;(\overline{X}_{(k)}-\overline{X}_{(k-1)})/\sqrt{2}\;S)$$

$$\text{PCS} \geq \int_{-\infty}^{\infty} \Phi^{k-1}\,[x+\sqrt{2}\;h_\nu\;(\sqrt{n}(\overline{X}_{(k)}-\overline{X}_{(k-1)})/\sqrt{2}\;\;S)]\;d\,\Phi(x) \tag{3.4}$$

where the non-negative function $h_\nu(t)$ is given by

$$\int_0^\infty [\Phi(h_\nu(t)-tu) + \Phi(-h_\nu(t)-tu)]\;d\,Q_\nu(u) = \alpha \tag{3.5}$$

for $t \geq t_{\alpha/2}(\nu)$ and $h_\nu(t) = 0$ for $0 \leq t \leq t_{\alpha/2}(\nu)$. Here, $t_{\alpha/2}(\nu)$ is the upper $\alpha/2$ quantile of the t-distribution with ν degrees of freedom and $Q_\nu(u)$ is the cdf of $\chi/\sqrt{\nu}$.

The values of the function $h_\nu(t)$ are given in Tables 1 and 2 for $\alpha = 0.05$, 0.10 and for selected values of ν and $t \geq t_{\alpha/2}(\nu)$. Note that $h_\nu(t) = h(t)$ for $\nu = \infty$. Details of the computational techniques are given in the Appendix. As can be seen from Figure 1, our computations have indicated that the function

$h_\nu(t)$ becomes nearly linear for moderately large values of $t-t_{\alpha/2}(\nu)$. For t values larger than those in Tables 1 and 2, the values of $h_\nu(t)$ satisfying (3.5) can be found numerically or be approximated by linear extrapolation. Especially in the case of known variance ($\nu=\infty$), it can be easily shown that

$$\lim_{t\to\infty} (h(t)-t) = -z_\alpha,\ t - z_{\alpha/2} < h(t) < t - z_\alpha \quad \text{for } t > z_{\alpha/2}. \qquad (3.6)$$

Figure 1 approximately here

It follows from the lower bound on $h(t)$ in (3.6) that the lower confidence bound in (3.2) is sharper than the one in Anderson, Bishop, and Dudewicz (1977). It can also be easily observed that in the special case of $k=2$ it reduces to the one in Faltin (1980).

Tables 1 and 2 approximately here

4. AN EXAMPLE

For illustration purpose, we consider an example given by Kleijnen, Naylor, and Seaks (1972), in which a firm that produces a single product from a multi-stage production process is interested in selecting the one most profitable production plan among $k=5$ possible plans. They run simulation experiments with a sample of size $n=50$ for each plan and assume that the profit using each plan has a normal distribution with a common unknown variance. The data are as follows:

Plan i	Mean profit	Standard deviation
1	2976.40	175.83
2	2992.30	202.20
3	2675.20	250.51
4	3265.30	221.81
5	3131.90	277.04

From the given data, plan 4 yields the largest sample mean and is selected as the most profitable plan. A reasonable question is: what kind of confidence statement can be made regarding the PCS? First, we observe that the pooled sample standard deviation is $s = 228.26$ with $\nu = 5(49) = 245$ degrees of freedom and $t = \sqrt{n}\,(\bar{x}_{(5)}-\bar{x}_{(4)})/\sqrt{2}\,s = 2.92$. Choosing $\alpha = .10$, we find $\sqrt{2}\,h_{\nu}(t) = 2.32$ by (3.6). Using Table A.2 in Gibbons, Olkin, and Sobel (1977) for the integral value in (3.4), we can state with 90% confidence that PCS $\geq .856$.

5. CONCLUDING REMARKS

The results in Section 2 are derived for location parameter families. However, similar results for scale parameter families can be obtained. For the problem of selecting the population with the largest scale parameter θ_i, the PCS in (1.1) is replaced by

$$\text{PCS} = \int_0^\infty \prod_{i=1}^{k-1} F(\theta_{[k]}y/\theta_{[i]})\, d\,F(y) \tag{5.1}$$

where $F(y/\theta_i)$ is the cdf of an appropriate statistic $Y_i \geq 0$. Similar analysis yields, under the assumption of MLR of the pdf $\frac{1}{\theta} f(\frac{y}{\theta})$ in y and θ, that the $100(1-\alpha)\%$ lower confidence bound in (2.4) can be replaced by

$$\hat{P}_L = \int_0^\infty F^{k-1}(y\, L(Y_{(k)}/Y_{(k-1)}))\, d\,F(y). \tag{5.2}$$

The function L(t) in (5.2) is defined by

$$H(L(t)/t) + H((tL(t))^{-1}) = \alpha$$

for $t \geq x_{\alpha/2}$ and $L_{(t)} = 1$ for $0 < t < x_{\alpha/2}$ where H(x) is the cdf of Y_1/Y_2 for $\theta_1 = \theta_2$ and $x_{\alpha/2}$ is the upper $\alpha/2$ quantile of H(x). Also, obvious modifications can be made for the problem of selecting the population with the smallest scale parameter. Such modifications can be useful, for example, for the normal variances problem.

As a final remark, we point out that the lower confidence bound in (2.4) is conservative due to the use of the inequality (2.1). To obtain an exact lower confidence bound on PCS, one needs simultaneous lower confidence bounds on $\theta_{[k]}-\theta_{[i]}$, i=1,2,...,k-1 which the author was unable to obtain.

APPENDIX

To show the existence of a non-negative function L(t) satisfying (2.2), we assume that log f(y) is concave and let $\Psi_t(a) = H(a-t) + H(-a-t)$ for fixed $t \geq x_{\alpha/2}$. Then

$$\frac{d}{da}\Psi_t(a) = H'(a-t) - H'(a+t)$$

$$= \int_{-\infty}^{\infty} [f(y-t)-f(y+t)]\, f(y-a)dy \tag{A.1}$$

where H'(x) is the pdf of H(x).

Note that the expression in the brackets in (A.1) changes sign once from - to + as y varies from $-\infty$ to $+\infty$. Therefore, by the sign diminishing property of MLR (see, for example, Lehmann 1954, p. 74), $\frac{d}{da}\Psi_t(a)$ changes sign at most

once from - to + as a varies from $-\infty$ to $+\infty$. Furthermore, by the symmetry of $H'(t)$, $\frac{d}{da}\Psi_t(a) = 0$ for $a = 0$. Thus $\Psi_t(a)$ is strictly increasing in $a \geq 0$. Also it can be observed that for fixed $t \geq x_{\alpha/2}$, $\Psi_t(0) = 2H(-t) \leq \alpha$ and $\Psi_t(a) \to 1$ as $a \to \infty$. Hence $L(t)$ can be defined by (2.2).

For constructing Tables 1 and 2, numerical evaluation of the integral in (3.5) was done via IMSL's subroutine MDTN. In the case of a known variance ($\nu=\infty$), MDNOR was used. The value of $h_\nu(t)$ was found numerically by finding a root of (3.3) or (3.5) via the modified regula falsi method with the accuracy up to 10^{-5}. Then, the values of $h_\nu(t)$ were rounded.

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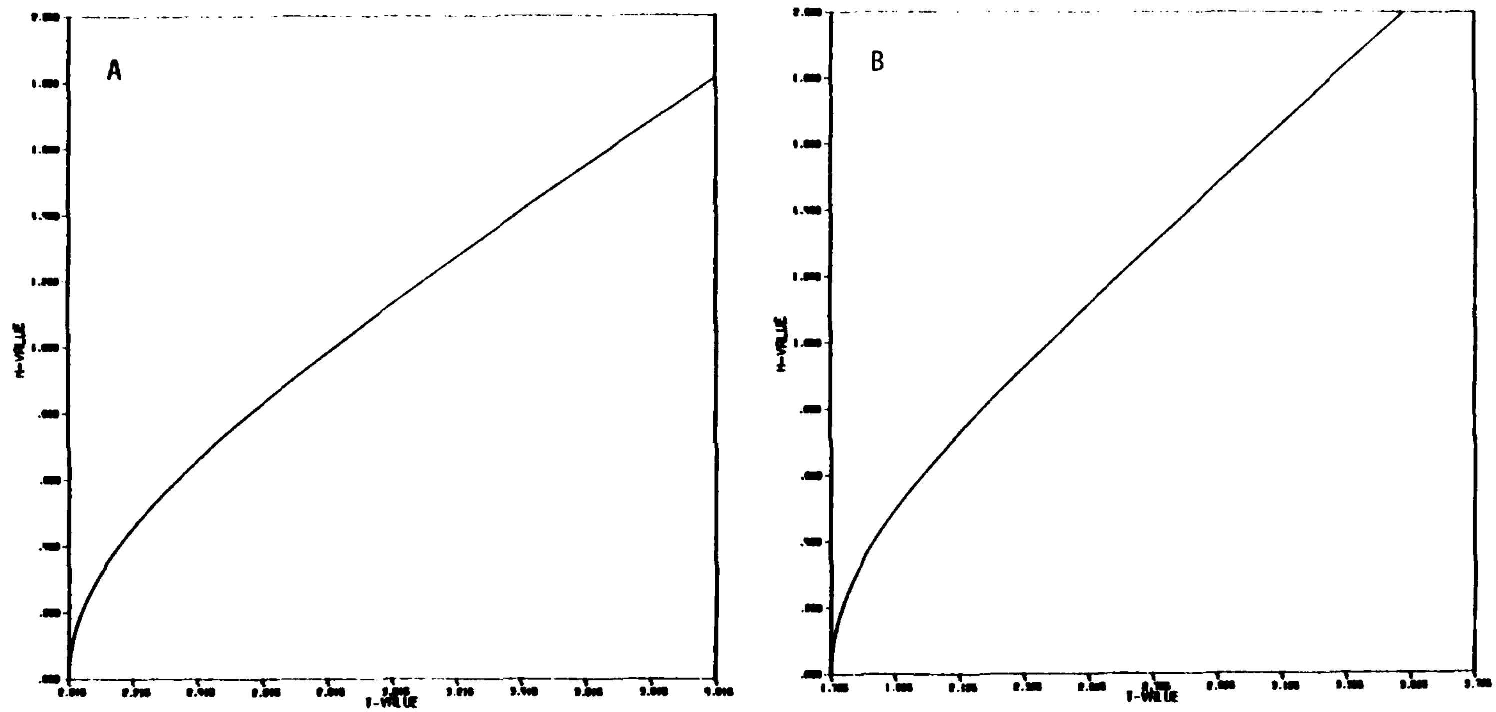

Figure 1. Values of $h_{\nu}(t)$ versus t values for α=.10. A: ν=5, B: ν=20

Table 1. Values of $h_\nu(t)$ for α=.05.

	$t - t_{\alpha/2}(\nu)$									
ν	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
5	.198	.282	.348	.405	.456	.502	.546	.588	.628	.666
6	.203	.290	.358	.416	.469	.517	.563	.606	.648	.688
7	.207	.295	.364	.424	.478	.528	.575	.619	.663	.704
8	.210	.299	.369	.430	.485	.536	.584	.630	.674	.717
9	.212	.302	.373	.435	.490	.542	.591	.638	.683	.726
10	.214	.305	.376	.439	.495	.547	.597	.644	.690	.734
11	.215	.307	.379	.442	.498	.551	.601	.649	.696	.741
12	.216	.308	.381	.444	.501	.555	.605	.654	.700	.746
13	.217	.310	.383	.446	.504	.558	.609	.657	.705	.751
14	.218	.311	.384	.448	.506	.560	.611	.661	.708	.755
15	.219	.312	.386	.450	.508	.562	.614	.664	.711	.758
16	.219	.313	.387	.451	.510	.564	.616	.666	.714	.761
17	.220	.314	.388	.453	.511	.566	.618	.668	.717	.764
18	.220	.314	.389	.454	.513	.568	.620	.670	.719	.767
19	.221	.315	.390	.455	.514	.569	.621	.672	.721	.769
20	.221	.316	.390	.456	.515	.570	.623	.674	.723	.771
30	.223	.319	.395	.461	.522	.578	.632	.684	.734	.783
60	.226	.323	.400	.467	.528	.586	.641	.694	.746	.796
120	.227	.325	.402	.470	.532	.590	.645	.699	.752	.803
∞	.228	.326	.404	.473	.535	.594	.650	.704	.758	.810

	$t - t_{\alpha/2}(\nu)$									
ν	.60	.70	.80	.90	1.0	1.1	1.2	1.3	1.4	1.5
5	.740	.810	.878	.944	1.009	1.072	1.134	1.196	1.256	1.317
6	.766	.840	.912	.982	1.051	1.118	1.185	1.251	1.317	1.381
7	.785	.862	.937	1.010	1.082	1.153	1.224	1.293	1.362	1.431
8	.799	.879	.956	1.032	1.107	1.181	1.254	1.327	1.399	1.470
9	.811	.892	.972	1.050	1.127	1.203	1.279	1.354	1.428	1.502
10	.820	.903	.985	1.065	1.144	1.222	1.299	1.376	1.453	1.529
11	.828	.912	.995	1.077	1.157	1.237	1.316	1.395	1.473	1.551
12	.834	.920	1.004	1.087	1.169	1.250	1.331	1.411	1.491	1.570
13	.840	.927	1.012	1.096	1.179	1.262	1.343	1.425	1.506	1.587
14	.845	.933	1.019	1.104	1.188	1.271	1.354	1.437	1.519	1.601
15	.849	.938	1.025	1.111	1.196	1.280	1.364	1.448	1.531	1.614
16	.853	.942	1.030	1.117	1.202	1.288	1.373	1.457	1.541	1.625
17	.856	.946	1.035	1.122	1.209	1.295	1.380	1.466	1.551	1.635
18	.859	.950	1.039	1.127	1.214	1.301	1.387	1.473	1.559	1.644
19	.862	.953	1.042	1.131	1.219	1.306	1.393	1.480	1.566	1.652
20	.864	.956	1.046	1.135	1.223	1.311	1.399	1.486	1.573	1.660
30	.880	.974	1.068	1.160	1.252	1.344	1.436	1.527	1.618	1.708
60	.896	.994	1.091	1.187	1.283	1.379	1.474	1.570	1.665	1.760
120	.904	1.004	1.102	1.201	1.299	1.397	1.494	1.592	1.690	1.787
∞	.913	1.014	1.115	1.215	1.315	1.415	1.515	1.615	1.715	1.815

Table 2. Values of $h_\nu(t)$ for α=.10.

	$t - t_{\alpha/2}(\nu)$									
ν	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
5	.224	.318	.392	.455	.512	.564	.613	.660	.704	.747
6	.228	.324	.400	.464	.522	.576	.626	.674	.720	.765
7	.231	.329	.405	.471	.530	.584	.636	.685	.732	.777
8	.233	.332	.409	.475	.535	.591	.643	.693	.740	.787
9	.235	.334	.412	.479	.540	.596	.648	.699	.747	.794
10	.236	.336	.415	.482	.543	.599	.653	.704	.753	.800
11	.237	.338	.417	.485	.546	.603	.656	.708	.757	.805
12	.238	.339	.418	.487	.548	.605	.659	.711	.761	.809
13	.239	.340	.420	.488	.550	.608	.662	.714	.764	.813
14	.240	.341	.421	.490	.552	.610	.664	.716	.767	.816
15	.240	.342	.422	.491	.553	.611	.666	.718	.769	.818
16	.241	.343	.423	.492	.555	.613	.668	.720	.771	.820
17	.241	.343	.424	.493	.556	.614	.669	.722	.773	.823
18	.242	.344	.424	.494	.557	.615	.670	.723	.774	.824
19	.242	.345	.425	.495	.558	.616	.671	.725	.776	.826
20	.242	.345	.426	.495	.558	.617	.673	.726	.777	.827
30	.244	.348	.429	.500	.563	.623	.679	.733	.786	.837
60	.246	.351	.433	.504	.569	.629	.686	.741	.794	.846
120	.247	.352	.435	.506	.571	.632	.689	.744	.798	.851
∞	.248	.354	.436	.508	.574	.635	.693	.748	.802	.856

	$t - t_{\alpha/2}(\nu)$									
ν	.60	.70	.80	.90	1.0	1.1	1.2	1.3	1.4	1.5
5	.830	.908	.984	1.058	1.131	1.202	1.272	1.342	1.410	1.479
6	.850	.932	1.011	1.089	1.165	1.240	1.314	1.387	1.460	1.532
7	.865	.949	1.031	1.111	1.190	1.268	1.345	1.421	1.497	1.572
8	.876	.962	1.046	1.128	1.209	1.289	1.369	1.448	1.526	1.604
9	.885	.972	1.058	1.142	1.225	1.307	1.388	1.469	1.549	1.629
10	.892	.981	1.068	1.153	1.237	1.321	1.404	1.486	1.568	1.650
11	.898	.988	1.076	1.162	1.248	1.333	1.417	1.501	1.584	1.667
12	.903	.994	1.083	1.170	1.257	1.343	1.428	1.513	1.598	1.682
13	.907	.999	1.088	1.177	1.264	1.351	1.438	1.524	1.609	1.694
14	.911	1.003	1.093	1.183	1.271	1.359	1.446	1.533	1.619	1.705
15	.914	1.007	1.098	1.188	1.277	1.365	1.453	1.541	1.628	1.715
16	.917	1.010	1.102	1.192	1.282	1.371	1.460	1.548	1.636	1.724
17	.919	1.013	1.105	1.196	1.286	1.376	1.465	1.554	1.643	1.731
18	.921	1.015	1.108	1.200	1.290	1.381	1.470	1.560	1.649	1.738
19	.923	1.018	1.111	1.203	1.294	1.385	1.475	1.565	1.655	1.744
20	.925	1.020	1.113	1.206	1.297	1.388	1.479	1.570	1.660	1.750
30	.936	1.033	1.129	1.224	1.319	1.413	1.506	1.600	1.693	1.786
60	.948	1.047	1.146	1.243	1.340	1.437	1.534	1.631	1.727	1.824
120	.954	1.054	1.154	1.253	1.352	1.450	1.549	1.647	1.745	1.843
∞	.959	1.061	1.162	1.263	1.363	1.463	1.563	1.663	1.763	1.863

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In the Problem of selecting the best of k populations, a natural rule is to select the population corresponding to the largest sample value of an appropriate statistic. As a retrospective analysis, a lower confidence bound on the probability of a correct selection is derived when the probability density function has the monotone likelihood ratio property under the location parameter setting. The result is applied to the normal populations with both known and unknown common variance. Tables to implement the confidence bound are provided.

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